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IN THE BEGINNING

F. BURTON JONES
Department of Mathematics, University of California, Riverside, CA 92521, USA

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The purpose of this paper is to discuss some of Bing's early work in continuum theory—the work he did at the University of Texas before he went to Wisconsin.

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simple web
pseudo-arc
simple closed curve
the 2-sphere
homogeneity

In the late spring of 1942 I left the University of Texas to do war work and Bing took my place. When I returned thirty months later he had almost completed his doctoral research. In fact, in the spring of 1945 he defended his thesis. In it he obtained a remarkable result—one that I could hardly believe and still find hard to believe.

Bing's Simple Web Theorem. *In order that a plane continuum be a simple web it is necessary and sufficient that it remain connected and locally connected upon the removal of any countable subset.*

What is a simple web? A continuum M is a *simple web* if it has two upper semicontinuous monotone decompositions $\mathcal{G}$ and $\mathcal{H}$ such that $M/\mathcal{G}$ and $M/\mathcal{H}$ are dendroids and for $G \in \mathcal{G}$ and $H \in \mathcal{H}$, $G \cap H$ is nonvoid and totally disconnected. It isn't hard to believe that the complement of a countable set in a simple web is connected because it intersects only countably many elements of $\mathcal{G}$ or $\mathcal{H}$. Moore [5] had been able to prove that a simple web was locally connected. But conversely to start out with a locally connected continuum and construct those decompositions just knowing that no countable set separated it, even if you knew additionally that no countable set would separate a connected open set, still seems to me to be a hard job; see [1] for details.

At the time of Pearl Harbor (circa 1941) I was refereeing a paper by Dick Wick Hall on the Kline 2-sphere conjecture. He had solved a fairly useful special case and I had some brief hope of being able to make his methods work for the whole thing. I don't remember whether I suggested this problem to Bing or not, but it would have been natural to have done so in view of his thesis research especially since I had given up on it and was working on other things. In any case, we discussed it.

Kline had conjectured that a locally connected, nondegenerate continuum which is separated by each of its simple closed curves but by no pair of its points would have to be a 2-sphere. This conjecture had been around for quite some time but Bing [2] proved it rather quickly.

Moise, having returned from the war, was busy proving that his pseudo-arc was an indecomposable plane continuum which was homeomorphic to each of its nondegenerate subcontinua. During the defense of his thesis the question was raised whether the pseudo-arc was homogeneous. Moise thought not, saying there were some points which were "hopelessly in the middle". Afterward Bing got to thinking about this question and proved that the pseudo-arc really was homogeneous; see [4]. This problem was twenty-five years old. So it began to be said that Bing was settling all the hard problems in topology. And indeed, he was making a good start. In fact, before working on the Kline 2-sphere problem, Bing [3] settled a question raised by Wilder in 1931: Suppose that a locally connected metric space M is the union of two subsets M_1 and M_2 such that each is irreducibly connected from the point a to the point b and such that $M_1 \cap M_2 = \{a, b\}$. Is M necessarily a simple closed curve? Bing answered this question "no" by constructing a collection $\mathcal{G}$ of point sets filling up the plane closed square disk S such that (1) for some pair of points, a and b, each element of $\mathcal{G}$ is irreducibly connected from a to b, (2) if $G, H \in \mathcal{G}$, then $G \cap H = \{a, b\}$, and (3) if $G, H \in \mathcal{G}$, then $G \cup H$ is locally connected and dense in S.

The high dimensional hereditarily indecomposable continua came later. He was not able to settle the *Normal Moore Space Metrization Problem* but this may very well not be a problem in topology. However, he got to thinking about metrization and later discovered the metrization theorem that bears his name.

In the late 1930s Professor Ingraham wrote R.L. Moore asking that he send Wisconsin one of his students like G.T. Whyburn. I remember Moore snorted that this was asking for a lot. But I think when Bing went to Wisconsin Moore felt that Ingraham would be satisfied. Certainly Bing had had a pretty good beginning.

References

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